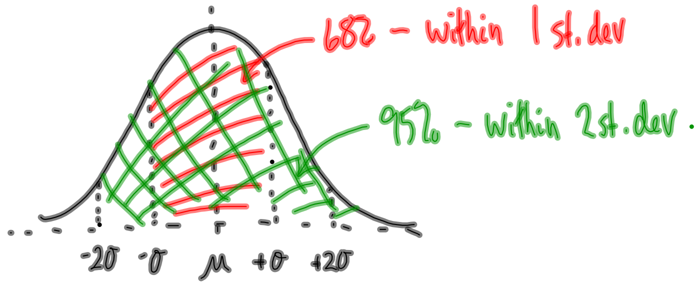
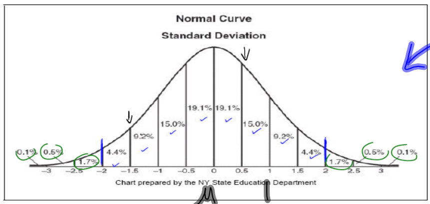
Normal Curve + Area under the curve

Recall from Math 10:





more precise.....
given to $\frac{1}{2}$ st.dev

μ divide this into 100 little strips!

What do we do when we don't have a value that is in increments of $\frac{1}{2}$ st.dev.??

You will be getting a chart that allows you to find the area under the normal curve in increments of 0.01 st.dev!!

This chart/table is referred to as z-scores

$$z = \frac{x-\mu}{\sigma}$$

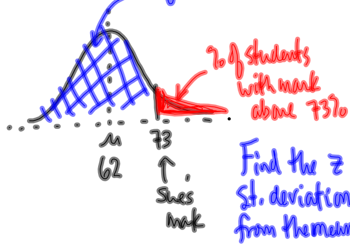
This gives us the # of st. deviations away from the mean

If $x < \mu$ then z is negative (below μ)

If $x > \mu$ then z is positive (above μ)

Z-Scores

Example - The average final mark on the NS English Exam was 62% with a standard deviation of 12%. The marks were normally distributed. Sue got a mark of 73%. What percentage of students scored higher than Sue on this exam?



Find the z-score (i.e. how many st. deviations Sue's mark is away from the mean)

$$Z = \frac{x - \mu}{\sigma}$$

$$Z = \frac{73 - 62}{12}$$

$$Z = \frac{11}{12}$$

$Z = 0.9166..$ or about 0.92.

(This means that Sue's mark is 0.92 of a st. deviation above the mean)

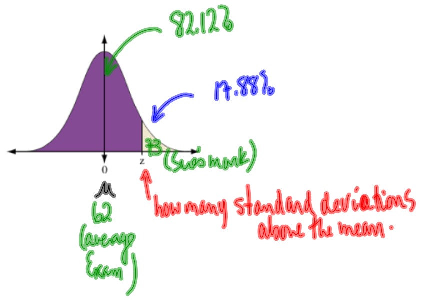
↑ round to 2 dec places!

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749

82.12% of the exam marks were below Sue's mark.

To get the % of students whose marks were above Sue's mark:

$$100\% - 82.12\% = 17.88\%$$



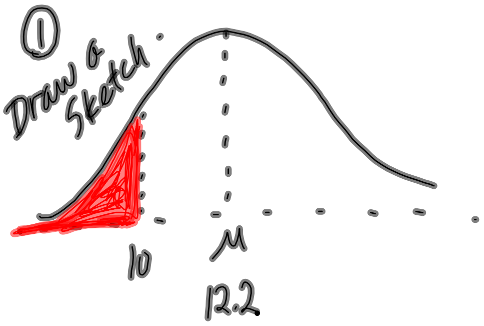
Example - The average life expectancy of a domestic short-haired cat (DSH) is normal with the given values:

$$N(12.2, 1.3) \Rightarrow N(\mu, \sigma)$$

normal μ σ

You got a new DSH kitten for Christmas. What is the probability that it will live for less than 10 years?

① Draw a Sketch.



② Find the z-score:

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{10 - 12.2}{1.3}$$

$$z = \frac{-2.2}{1.3}$$

$$z = -1.69$$

round to 2 dec places since the chart only goes to 2 dec.

③ Look up the z-score on chart.

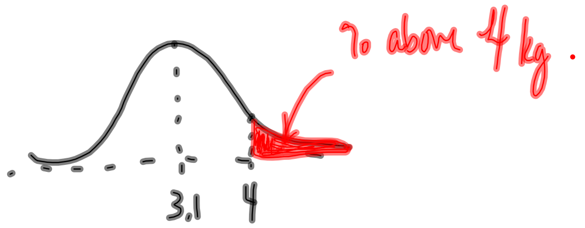
	0.09	0.08	0.07	0.06	0.05
−2.3	0.0084	0.0087	0.0089	0.0091	0.0094
−2.2	0.0110	0.0113	0.0116	0.0119	0.0122
−2.1	0.0143	0.0146	0.0150	0.0154	0.0158
−2.0	0.0183	0.0188	0.0192	0.0197	0.0202
−1.9	0.0233	0.0239	0.0244	0.0250	0.0256
−1.8	0.0294	0.0301	0.0307	0.0314	0.0322
−1.7	0.0367	0.0375	0.0384	0.0392	0.0401
−1.6	0.0455	0.0465	0.0475	0.0485	0.0495
−1.5	0.0559	0.0571	0.0582	0.0594	0.0606
−1.4	0.0681	0.0694	0.0708	0.0721	0.0735
−1.3	0.0823	0.0838	0.0853	0.0869	0.0885
−1.2	0.0985	0.1003	0.1020	0.1038	0.1056

4.55% of DSH cats will live less than 10 years.

Example - The following notation is given for weights of newborn babies. N(3.1kg, 0.4 kg). Your cousin just had a baby that weighed 4 kg (about 8lbs 13oz) Was this unusual?

normal, $\mu = 3.1$kg and $\sigma = 0.4$kg

① Sketch a graph:



② Find the z-score:

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{4-3.1}{0.4}$$

$$z = \frac{0.9}{0.4}$$

$$z = 2.25$$

③ Use the chart to find the % below (to the left) of the z-score.

	0.00	0.01	0.02	0.03	0.04	0.05	
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961

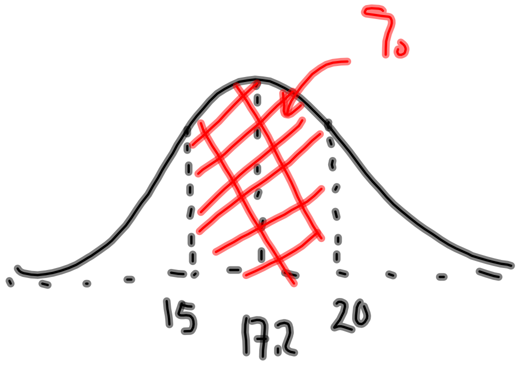
98.78% of newborns weigh less than 4kg

100% - 98.78% = 1.22%

↑ % that would weigh more than 4kg

This would "probably" be considered unusual

Example 4 Any defense player in the NHL will spend an average of 17.2 min on the ice per game. This is N(17.2, 2.2) What is the probability that your favorite player will spend between 15 and 20 min on the ice in his next game?

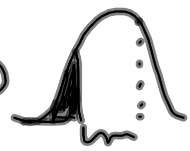


Find each z-score:

$$z = \frac{15-17.2}{2.2} = -1.00$$

$$z = \frac{20-17.2}{2.2} = \frac{2.8}{2.2} = 1.27$$

	0.09	0.08	0.07	6	5	4	3	2	1	0
−1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
−1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
−1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
−0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841



	0	1	2	3	4	5	6	7	8
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997

Between 15 + 20 minutes:

89.80% − 15.87% = 73.93%

So there is a 73.93% probability that he will play b/w 15 + 20 min